Theoretical Examination of the Absorption of Energy by Snow Crabs Exposed to Seismic Air-gun Pulses: Stage 2 - Improvements to Model and Examination of Resonances

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Executive Summary

This work carries on the work presented in a previous report [21, Lee-Dadswell, 2008]. It aims to evaluate the likelihood that seismic pulses could cause direct damage to the tissues of snow crabs. In the previous study it was shown that damage by heating of the tissues is extremely unlikely. The current focus is, therefore, on determining whether the particle displacements that occur inside the crab are sufficient to cause tearing/crushing of the tissues.

The calculations presented in this report improve on the previous calculations in two major ways.

1. The interior of the crab is treated as a solid with very low shear modulus. This is much more realistic than the previous calculations which treated the crab tissues as a liquid. It appears that this improvement to the model makes important differences to the outcome of the calculations, but does not change the overall conclusions.

2. We are now able to go beyond simply plotting displacements at a few frequencies. The computer application that we are building to carry out these calculations is now able to plot the resonance spectrum of the crab in a variety of ways. These plots reveal what frequencies of sound the model crab is likely to be most sensitive to.

The model of the interaction between the seismic pulse and the crab is still highly idealized. The seismic pulse is treated as a plane wave. The crab is treated as a thin elastic spherical shell containing a homogeneous elastic material. The elastic constants of the shell and inner (tissue) material are based on reports of these values from the literature. Classical acoustical theory yields a set of equations which can be solved for any sound frequency in order to determine the displacement amplitude at all points inside the model crab. The current calculations allow us to draw the following conclusions:

1. Treating the interior tissues as a solid with low shear modulus produces a large qualitative difference in the outcome of the calculation. The resonance spectrum of the sphere with the solid interior is extremely rich, containing many closely spaced, narrow resonances, compared with the sparse spectrum of the sphere with the liquid interior which contains only a few broad resonance peaks.

2. The amplitude of vibration inside the crab that results from the seismic pulse can be crudely estimated from the resonance spectrum simply by averaging the displacement amplitude over all frequencies. Despite the large qualitative difference which results from treating the interior of the crab as a solid, there appears to be little quantitative difference. Both models produce very similar estimates for the amplitude of vibration inside the crab.

3. The overall conclusion of the previous study was that the amplitude of vibration inside the crab was insufficient to directly cause tissue damage to the crab. This conclusion is unchanged by the present study. However, the present study significantly improves the confidence with which this conclusion can be stated.

4. The stresses at boundaries between different tissue masses can be surprisingly large. This backs up reports in the literature that the outer membranes of crab ovaries can become detached when the crabs are exposed to seismic testing. It is possible that similar large stresses could occur anywhere in the crab where two tissues with significantly different elastic properties are attached together. There is a strong possibility that this mechanism could cause direct physical damage to crabs.

There continues to be a need to experimentally determine some of the physical characteristics of crab tissues. What is particularly needed are measurements of the maximum fractional strain. Additionally, these theoretical investigations reported on there have proceeded to the point that experimental validation of these calculations is needed. This will make it easier to determine which of the approximations currently being made are the most significant and will help to direct future theoretical and experimental efforts.

Introduction

Offshore seismic surveys have many potential impacts on fisheries and on the environment in general. The sound generated during a seismic survey can be sufficiently loud (up to about 255 dB rel. 1 μPa [25, MacGillivray, 2005]) that various marine organisms may be affected. Considerable work has been done on the effects on marine mammals (e.g. [9, Goold, 1998]). Some work has also been done on fish which are vulnerable because of their air bladders. Comparatively little work has been done to determine the effects on marine invertebrates. At present in Nova Scotia there is particular concern that seismic surveys might have detrimental effects on snow crabs [3, Chadwick, 2004], [27, Moriyasu, 2004], [39, Walmsley, 2007].

The possible effects which are envisioned [39, Walmsley, 2007] range from subtle behavioural and environmental effects which might affect catchability, through intermediate behavioural effects which could affect reproductive or feeding success, through to immediate, direct physical damage. Previous experimental studies have been carried out [3, Chadwick, 2004] to try to assess whether physical damage was sustained by crabs in the viscinity of a seismic survey. However, these studies were largely inconclusive. Among the reasons for this are:

- The sound levels that the crabs were exposed to may have been below the levels at which physical damage might be expected.

- There is insufficient knowledge of what types of physical damage ought to be expected.

- There is insufficient knowledge of the physiology of a healthy crab for damage to be recognized when it has occurred.

- Differences between test and control groups may have been caused by factors other than exposure to seismic pulses [3, Chadwick, 2004].

An earlier theoretical study [21, Lee-Dadswell, 2008] concluded that even the highest intensity sound produced in a seismic survey is probably insufficient to cause direct physical damage in the form of tissue tearing or crushing. However, the displacements predicted might be sufficient to cause some of the subtle physical damage that was tentatively identified in [3, Chadwick, 2004]. However, the model used in that study was sufficiently crude that the confidence with which this conclusion could be stated was quite low.

Objectives

The previous report [21, Lee-Dadswell, 2008] contained the results of calculations using the simplest reasonable model for a crab exposed to high intensity sound.

1. The seismic pulse was treated as a plane wave. This is expected to be approximately valid for distances from the source that are large compared to the source size (i.e. in deep water).

2. The crab was treated as spherical.

3. The crab carapace was treated as a thin, uniform, elastic shell.

4. The interior of the crab was treated as a uniform compressible fluid.

5. No damping (viscosity effects) were included in the calculation.

6. The seabottom was not included in the model. This might be significant because of reflections from the bottom.

7. Linear acoustics was assumed to be valid.

This model has been used in various papers in the research literature seeking to model sound scattering by crustaceans [26, Matchlup, 1952], [34, Stanton, 1990], [35, Stanton, 2000]

The present study aimed to improve the model by relaxing one or more of the above approximations. The primary aim was to improve on point 4. by modeling the interior of the crab as an elastic solid with small shear modulus. This has the effect of including transverse waves in the crab's interior, which are ignored when the interior is treated as a fluid. It is expected that transverse waves are poorly transmitted into the interior of the crab and that this improvement should produce little change in the overall conclusions. Nevertheless, the assumption should be checked, particularly since transverse waves damp far more strongly in tissues and so this would be a far more efficient means for transferring energy from the seismic pulse to the crab.

A secondary objective was to begin to model a single organ with a surrounding membrane inside the crab. In particular, it is reported in [3, Chadwick, 2004] that separation of the outer membrane from the ovary was observed in crabs which had been exposed to seismic testing. Treating a crab's interior as nonuniform and including organs would be extremely difficult. Instead we will model an individual organ with a surrounding membrane, embedded in connective tissue as a sound wave moves through the surrounding tissue.

A significant additional objective of this study was to further develop the Maple code being used for these calculations with an eye towards eventually producing a suite of code which might serve as a useful tool for investigators needing to examine the interaction between sound and various objects. This would clearly have application well outside of studying the effects of seismic pulses on crabs.

Approach

There has been some work on modeling acoustic scattering due to various invertebrate (e.g. [26, Matchlup, 1952], [34, Stanton, 1990], [35, Stanton, 2000]) and vertebrate (e.g. [4, Clay, 1994], [23, Love, 1978], [6, Feuillade, 1998]) animals. In the works on invertebrates the animal is generally modeled as a thin shell (spherical, cylindrical or a bent cylinder) filled with a nonviscous fluid. However, the internal tissues of these animals are probably better modeled as a solid with a very low shear modulus [20, Lebedeva, 1965]. Also, these previous studies have focused on the scattering by these animals probably because the motivation behind the studies was prediction of the sonar signature of the animals. A principle goal of the current paper is to examine the amplitude of motion inside the shell and to examine the differences between the fluid filled shell model and a solid filled shell model.

A seismic pulse consists of all frequencies of sound. The spectrum has a large peak below 1000 Hz, and moderate spectral density up to 10-30 kHz [9, Goold, 1998]. This, then, is the range of frequencies that are of interest in this study. It is common in the acoustics literature to work in terms of the dimensionless quantity, ka, rather than in terms of frequency. In this notation $k = 2\pi/\lambda$, where λ is the wavelength and a is the size of the object that is interacting with the sound (in this case a would stand for the diameter of the crab). The "wave vector", k, can be thought of as a rescaled frequency. The reason for reporting results in terms of ka is that the response of an object to sound always depends more directly on ka than it does on frequency. If two objects of the same shape but different size are exposed to sounds at all frequencies then their responses will look different when plotted against frequency, but will look the same when plotted against ka. Accordingly, many results in this report will be presented in terms of ka since presenting them in this way makes them independent of the size of the crab. However, some results will be plotted against frequency. In these cases, the crab will be chosen to have a diameter of 10 cm. For a crab of 10 cm diameter, a frequency of 30 kHz corresponds roughly to $ka \simeq 5$.

Following earlier work [26, Matchlup, 1952], [34, Stanton, 1990], [8, Goodman, 1962], [10, Hasegawa, 1993] we adopt an exact approach from classical acoustics. The sound propagation obeys a well studied differential equation and we can write down the form of its solutions immediately. Accordingly we write down expressions representing the waves outside the crab, in the shell and in the crab's tissues. We then write down equations which express the conditions which must be obeyed by the waves at the water-shell boundary and at the shell-tissue boundary. This yields a system of linear equations which can be solved for any frequency of incoming sound. These solutions can be used to obtain the sound intensity at any point inside or outside the crab.

As a result of the previous study [21, Lee-Dadswell, 2008] a Maple sheet already existed which solved the differential equations for the fluid filled model. An undergraduate research assistant, Patrick Toupin, was hired to carry out various checks of the results from this Maple sheet. We checked that the solutions obeyed conservation of energy and that the conditions that were supposed to be

obeyed at the boundaries were rigorously obeyed. Additionally, the results of [8, Goodman, 1962] and [34, Stanton, 1990] were reproduced as a further check of the Maple sheet. This involved significant modification of the Maple sheet to be able to carry out the calculations repeatedly and tablulate the results. Patrick Toupin then made the necessary adjustments to the sheet so that it solved the differential equations for the case of a solid interior.

The physical characteristics of the tissues used in the previous study were mostly retained in this one. A consultation with Dr. Edwin DeMont yielded an additional source which gave additional information on the physical characteristics of crustacean cuticle [38, Wainwright, 1976]. Dr. DeMont additionally advised that, in his opinion, further effort on experimentally determining the physical characteristics of crab tissues is probably not warranted. The physical characteristics would be highly variable from crab to crab and from one part of an organ to another. Thus, the precision with which we know these physical characteristics would not be appreciably improved by further experiments. The exception is the elastic limits of tissues. These are so poorly known that further experiments *would* be of use.

A secondary goal in this study was to attempt to investigate the possibility of separation of membranes from organs. An organ, embedded in tissue, will have a soft solid region outside the organ, a very thin membrane of a somewhat stiffer solid, and an interior that is soft solid. We have made further adjustments to our Maple sheets to be able to handle a solid exterior region. Also, a good measure of whether tissue separation is likely ought to be the radial component of stress (force per unit area) at the boundary between the organ and its membrane. At this point we are able to generate plots of stress as a function of position on the membrane (angle from straight up). It remains as future work to be able to plot maximum stress as a function of frequency.
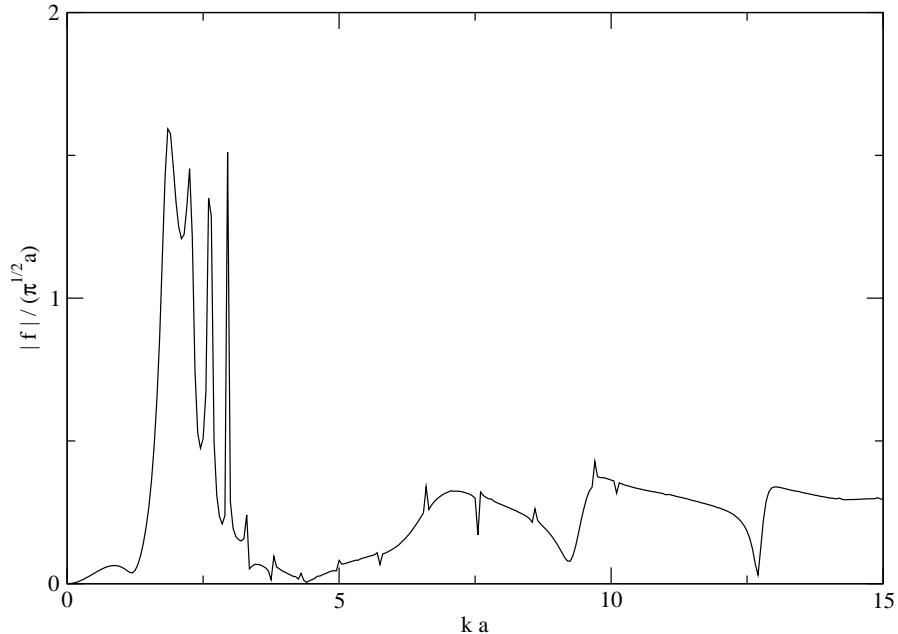
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Figure 1: Normalized back scatter amplitude vs. ka for an air filled, stainless steel, spherical shell with shell thickness $1/20$ of the shell radius. The highest modes used were $\ell = 14$. This should be compared with Fig. 8 (a) in [34, Stanton, 1990].

Findings

Identification of Resonances

Our Maple code, when applied to the problem of an elastic spherical shell with fluid interior (as in the previous study) produces results which are consistent with other work reported in the literature. We have used it to examine scattering of sound by a steel sphere in water, with water or air inside. The results of these calculations can be compared with published figures in [34, Stanton, 1990]. Following that source we calculate the "target strength" for backscattering by steel spheres. A sample plot of the target strength is shown in Fig. 1. We find excellent agreement between our plots and those of [34, Stanton, 1990] up to $ka \simeq 10$. Above $ka \simeq 10$ there are significant differences between our plots and those in [34, Stanton, 1990]. This is simply because we have not included as many terms in our sums over modes as was done in [34, Stanton, 1990]. However, since we are only interested in the frequency regime below $ka \simeq 5$, this is of no concern. If we wish to get better accuracy at high values of ka it is a simple matter of keeping more terms in our sums over modes.

While backscattering amplitude or "target strength" are of great interest in

the extensive literature related to remote sensing via sonar, the primary interest in this study is the amplitude of motion inside the crab. This can be obtained from the same calculations that give the backscattering strength. Plotting the amplitude of oscillations vs. frequency allows us to identify resonances of the model crab. With the differential equation solved we can calculate the amplitude of vibration at any point within the model crab. However, our primary interest in this study is the *maximum* amplitude of vibration within the crab. This will be more useful for assessing what frequencies are likely to be most dangerous to the crab. We can have the Maple sheet generate the amplitude at all points on a grid within the crab and then select the largest amplitude from these. We can do this for all frequencies and plot the result. This is shown for a model crab with fluid interior in Fig. 2. Note that at most frequencies the maximum amplitude of vibration inside the crab is quite small. However, there are distinct frequencies at which the amplitude is much larger. These are resonances of the model crab. If one were to expose the crab to high intensity sound at a resonance frequency this would result in severe damage to the crab. Fortunately, a seismic pulse is not like this. It is a mix of all frequencies and the intensity at any given frequency is not very high.

This can now be repeated for the model crab with solid interior. The result is shown in Fig. 3. The spectrum of resonances is extremely different when the interior is modeled as a solid. This is because the shear modulus of the interior is very low and as a result the speed of transverse waves inside the crab is very small. This produces a rich spectrum of closely spaced resonances. However, these resonances are mostly quite weak (low amplitude) and narrow (they are excited only by a narrow band of frequencies). This reflects the poor transfer of energy between the shell and tissue because of the enormous difference in shear modulus which was described in the earlier report [21, Lee-Dadswell, 2008]. These weak, narrow resonances will contribute little to the overall amplitude of vibration of the model crab when exposed to sound consisting of all frequencies.

Figure 3 does not give a full idea of the resonance spectrum because it contains such a dense forest of peaks that the details of individual peaks cannot be seen. We have used the Maple sheet to generate some similar plots over a narrower range of frequencies with higher resolution. An example of this is shown in Fig. 4. On this scale the structure of individual peaks is visible. There are clearly several different "types" of peaks. These correspond to modes of different symmetries. The appendix discusses how peaks can be assigned to their modes.

With the resonance spectrum obtained we can now get an approximate idea of the total amplitude of oscillation inside the crab. The methods for doing this are somewhat technical and involve some approximations. They are described in the Appendix but they amount approximately to averaging over all frequencies in the resonance spectrum. Doing this for the model with fluid interior we obtain an amplitude of 4.7, where given the approximations involved even quoting two significant figures is almost certainly overstating out precision. So the fluid filled model predicts that the maximum amplitude of oscillations inside the crab could be as great as 5 times the amplitude of oscillations outside the crab.
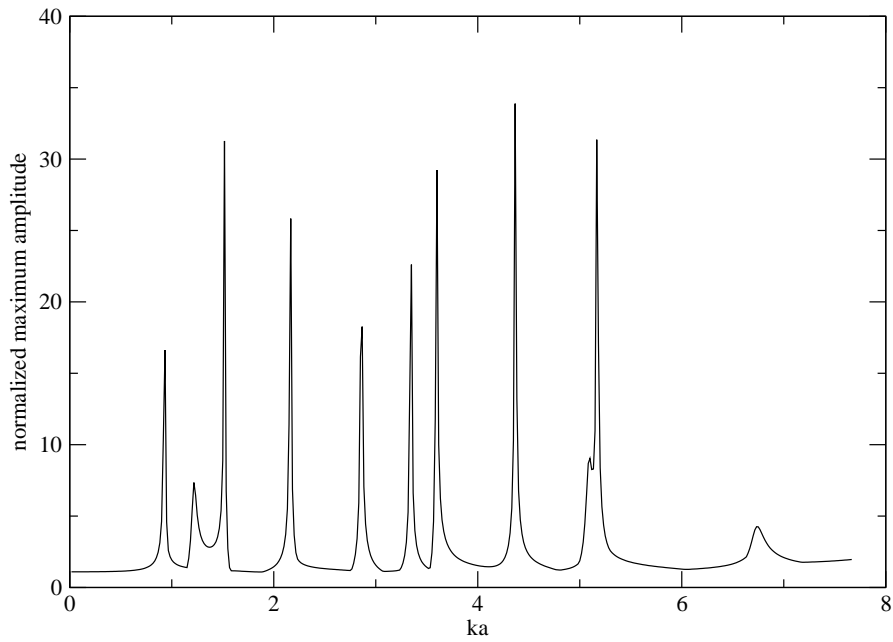
Figure 2: Maximum amplitude of particle motion inside shell normalized to amplitude of incoming wave for a shell with liquid interior. The horizontal axis is the dimensionless quantity, ka, which is essentially a rescaled frequency. For a 10 cm diameter crab, $ka \simeq 5$ corresponds to a frequency of 30 kHz which is the maximum frequency of interest in this study. The maximum amplitude is normalized to the amplitude of the incoming wave (i.e. an amplitude of 1 corresponds to the same amplitude as that of the incoming wave).
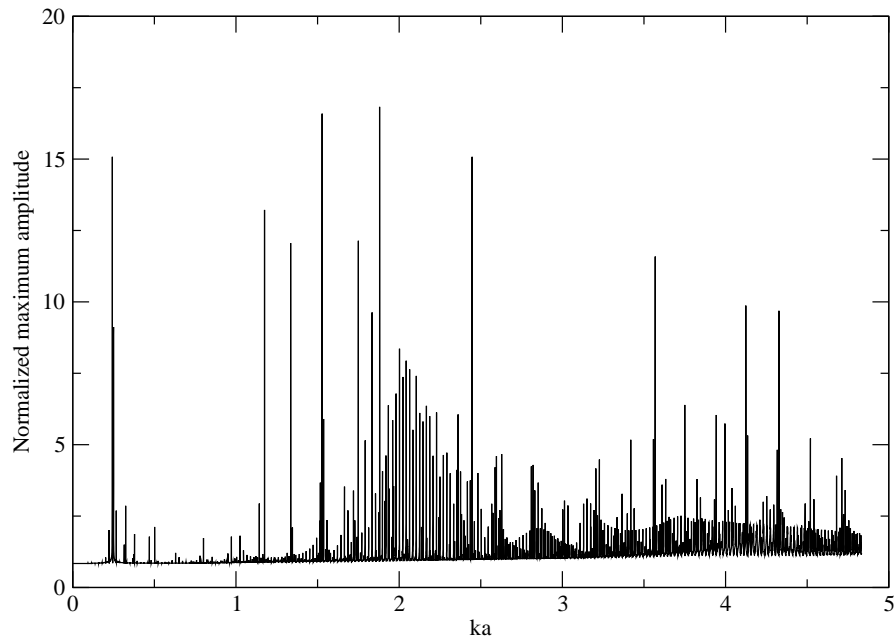
Figure 3: Maximum amplitude of particle motion inside shell normalized to amplitude of incoming wave for a shell with solid interior. The horizontal axis is the dimensionless quantity, ka, which is essentially a rescaled frequency. For a 10 cm diameter crab, $ka \simeq 5$ corresponds to a frequency of 30 kHz which is the maximum frequency of interest in this study. The maximum amplitude is normalized to the amplitude of the incoming wave (i.e. an amplitude of 1 corresponds to the same amplitude as that of the incoming wave).
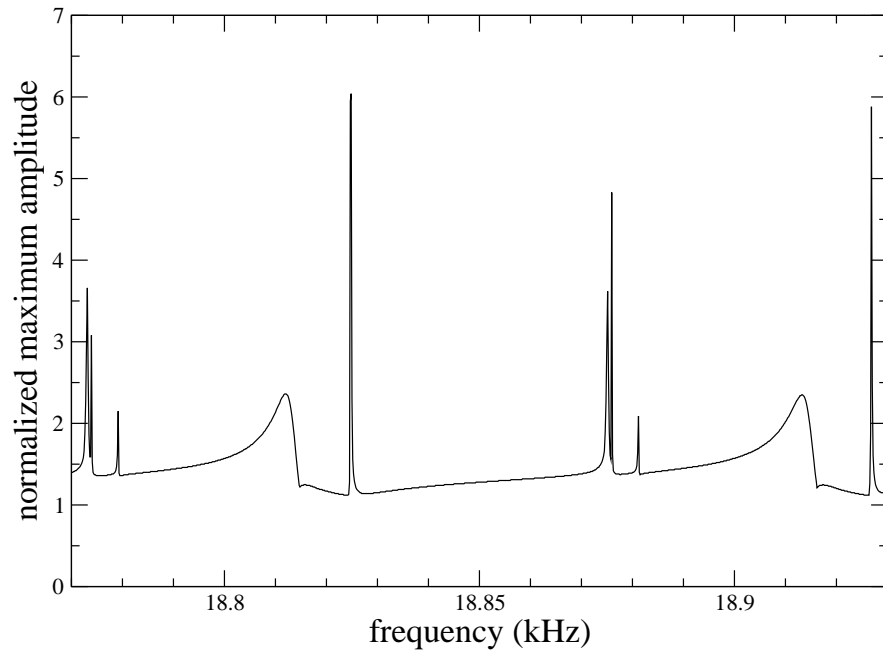
Figure 4: Normalized maximimum amplitude vs. frequency over a narrow range of frequencies for the model with solid interior. The structure of individual peaks is visible at this scale.

Repeating this with the solid filled model the resulting prediction for the maximum amplitude of oscillation is approximately 2.5 times the amplitude of oscillations outside the crab. Once again, this almost certainly overstates the precision and should be taken as only an order of magnitude estimate. What is clear is that the inclusion of transverse waves in the tissues by modeling the tissues as a solid does not make an appreciable difference in the estimate of the overall amplitude of motion inside the crab. As was reported in [21, Lee-Dadswell, 2008], to exceed the elastic limits of the tissues the oscillations would have to be approximately 50 times the amplitude outside the crab. So, whether the crab interior is modeled as a fluid or a solid, the predicted amplitude of motion is about a factor of 10 below the threshold at which tearing or crushing would be expected.

A significant result of these calculations is that the amplitude of shear waves in the tissues is usually of the same order of magnitude as the amplitude of longitudinal waves. This is included in the calculation of the amplitude of oscillation reported above. It also allows us to check whether there is risk of heating to the tissues due to absorption of transverse waves via viscous damping. The coefficient of absorption for transverse waves is *much* larger than the coefficient of absorption for longitudinal waves. In [21, Lee-Dadswell, 2008], the coefficient of absorption for longitudinal waves was estimated to be in the range of 1×10^{-7} to 1×10^{-5} m^{-1}. In other words a longitudinal wave through tissue damps to half of its original intensity over a distance of several thousand km. In contrast, the coefficient of damping for transverse waves is of order 10 m^{-1} (i.e. a transverse wave traveling through tissue damps to half of its original intensity over a distance of only about 10 cm). This is about 10^6 times that of longitudinal waves. So damping of transverse waves through tissues is quite strong. If the intensity of transverse waves through the crab tissues was very high then this would be a serious concern. However, while the amplitude of shear waves is similar to that of longitudinal waves, the intensity is not since the wave intensity is proportional to the wave speed. The speed of transverse waves is about 1/100 of the speed of longitudingal waves and so the intensity is also about 1/100 that of longitudinal waves. Combining the much higher damping coefficient with the much lower intensity, the crab absorbs about 10^4 times as much energy from the shear waves as it does from the longitudinal waves. The absorption of longitudinal waves was estimated in [21, Lee-Dadswell, 2008] to result in a temperature increase of the crab of about 1×10^{-7} C. So the absorption of transverse waves should raise the temperature of the crab by about 1×10^{-3} C (one thousandth of a degree). While this is much larger, it is still of no concern. This confirms the assumption made in [21, Lee-Dadswell, 2008] that transfer of transverse waves from the shell to the tissue is very weak because of the enormous difference in transverse sound speed in the two media.

Stress at the Membrane Boundary of an Internal Organ

We are able to examine plots and animations of stress vs. position on the membrane at various frequencies. Several example plots are shown. The stress
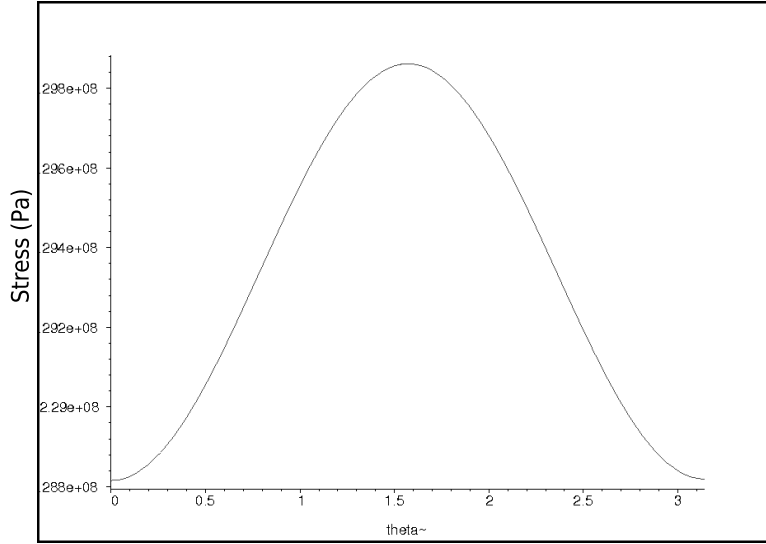
Figure 5: Stress as a function of position (angle) for frequency 5000 Hz on the membrane surface. theta=0 corresponds to the top of the organ and theta=$\pi \simeq$ 3.14 corresponds to the bottom of the organ.

as a function of position shows the expected behaviour:

- It is an oscillating function.

- At low frequencies only a small portion of one oscillation is seen from top to bottom of the organ. But at higher frequencies a full oscillations can be seen over the height of the organ. At frequencies higher than those examined several full oscillations would occur between the top and bottom of the organ.

- In animations it can be seen that the locations of maximum stress move downward on the organ.

These plots were generated using an incident amplitude of particle oscillations of 0.2 mm, which is approximately correct for the maximum intensities of sound to be expected [21, Lee-Dadswell, 2008]. The incident wave is assumed to be a traveling, plane, longitudinal wave. This is approximately valid at frequencies which are not resonant frequencies for the crab, where the sound wave essentially just travels through the crab with little distortion.

A disturbing outcome of examining the graphs is the large size of the stresses exerted at the organ membrane boundary. Even at relatively low frequencies (1000 Hz) the maximum stress tends to be of the order of 10^7 Pa (ten million Pascals). At higher frequencies the maximum stress increases so that by 30 000 Hz the maximum stress can be of order 10^9 Pa (a billion Pascals). Given that
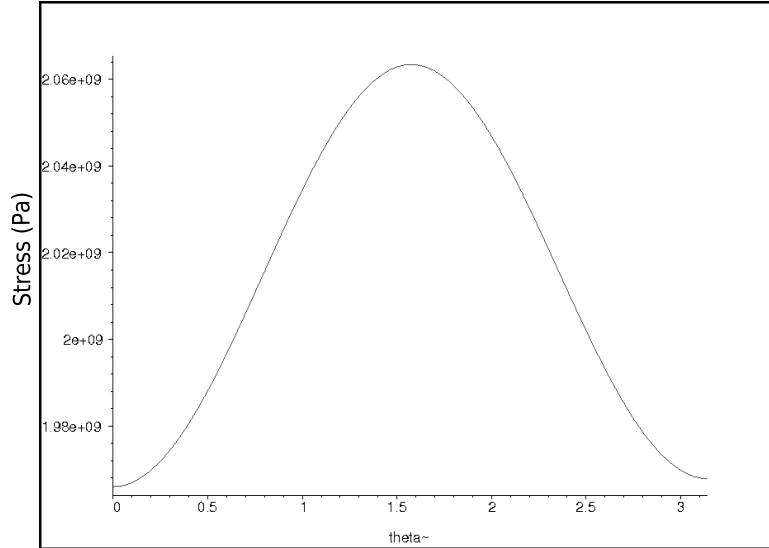
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Figure 6: Stress as a function of position (angle) for frequency 15000 Hz on the membrane surface. theta=0 corresponds to the top of the organ and theta=$\pi \simeq$ 3.14 corresponds to the bottom of the organ.

the ultimate strength of most tissues is only of the order of 10^6 Pa or sometimes 10^7 Pa, this is of serious concern.

The above should be interpreted in light of the fact that the model incoming sound is of a single frequency. The actual incoming sound from a seismic test contains all frequencies, and the amplitude at any given frequency is quite low. The actual stresses experienced would be a sum of many much smaller contributions from all frequencies. But given that even low frequencies yield such high stresses suggests that the stresses caused by the loudest sounds expected could easily exceed 10^7 Pa. This should be more than enough to cause separation of the membranes as reported in [3, Chadwick, 2004]. This raises quite a number of questions. Among them are:

- How harmful is this membrane separation? It is reported in [3, Chadwick, 2004] that no differences in reproductive viability were observed between experiment and control groups. However, as is noted in that study, the experiment and control groups may not have been comparable.

- Why would the ovaries be particularly succeptible? Does this happen to other organs? Could such damage to other organs have simply been missed in the lab work?

- Could the observed bruising of the hepatopancreas in [3, Chadwick, 2004] be related to this? The hepatopancreas is a highly inhomogeneous organ.
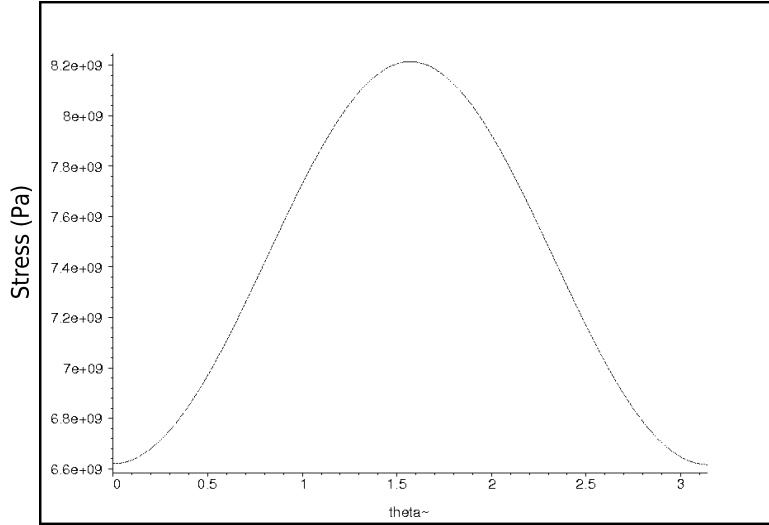
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Figure 7: Stress as a function of position (angle) for frequency 30000 Hz on the membrane surface. theta=0 corresponds to the top of the organ and Theta=$\pi \simeq$ 3.14 corresponds to the bottom of the organ.

> The variations in tissue properties within the organ may cause stresses similar to those at boundaries between tissue masses.

In addition to the above, where the incoming wave is a traveling wave, we have also been able to modify the Maple sheets to treat the case where the incident wave is a plane, longitudinal *standing* wave. This is approximately valid at low frequencies that are resonant frequencies of the crab. Similar patterns are seen to those reported above in terms of the maximum stresses observed.

Animations Available for Viewing

The development of much of our knowledge about relative amplitudes of vibration, resonance vs. non-resonance frequencies and characteristics of different vibrational modes has been greatly aided by examining animations of the particle displacements. With current computing power it takes several minutes for our Maple sheets to generate one of these animations. Several of these animations are included on a CD which accompanies this report.

Recommendations and Conclusions

This study largely upholds the conclusions of the previous report [21, Lee-Dadswell, 2008]; seismic testing is unlikely to directly damage the bulk tissues of crabs. The inclusion of transverse waves in the crab tissues does not change this conclusion in any way. However, a significant result of this study is that interfaces between tissues, where the tissues in question have different elastic properties, can experience significant stresses. In particular:

- Modeling the crab's interior as an easily sheared solid rather than as a liquid causes large, and possibly important, qualitative differences in the spectrum of resonances of the model crab. However, it does not cause a significant quantitative difference in the predicted amplitude of oscillations inside the crab. Thus, the risk of damage to tissues due to exceeding their elastic limits does not seem to be affected by the presence of transverse waves in the tissues.

- Absorption of transverse waves in the tissues is much stronger than the absorption of longitudinal waves. However, transverse waves are so poorly transmitted into the tissues that this should not lead to appreciable heating of the crab tissues.

- The apparent lack of data on physical characteristics of crab tissues may not be as serious an issue as previously reported. The large variations in these quantities means that experiments to determine them would be unlikely to yield considerably more precision in our knowledge of their values. The exception to this is the elastic limits of tissues, which are so poorly studied that experiments are warranted.

- Organs with "stiff" membranes surrounding them may suffer separation of the membranes from the organ due to large stresses at the interface between the organ and the membrane. Thus, the theory is consistent with the report in [3, Chadwick, 2004] that the outer membranes of ovaries became detached. Other tissues in which the elastic properties vary considerably over short distances may also be vulnerable to damage in this way. This might explain the report in [3, Chadwick, 2004] of bruising in the hepatopancreas.

The calculations in this report still contain a significant number of approximations. Among them:

- The presence of the sea bottom is not included in the model. This may have important effects because of the reflection of sound by the sea bottom. Solution of the differential equations with an additional boundary to represent the sea bottom presents significant challenges and will likely require a different theoretical approach. Additional investigation is probably warranted, but should be guided by experiment both because of the difficulty and because of the significant possibility that this issue is not important.

- The crab is still being modeled as a sphere. Modeling it as an oblate spheroid (flattened sphere) would be closer to reality.

- The effects of damping in tissues could be more accurately handled by including them in the solution to the differential equations describing the waves. This would make the calculations much more difficult and, given the weakness of damping of longitudinal waves and the low intensity of transverse waves, it is unlikely that including this in the model would change the overall conclusions in any way. Thus, this is probably of academic interest only.

- The tissues and shell are treated as homogeneous and isotropic (the same everywhere, and responding to disturbances in the same way, regardless of the direction of the disturbance). However, the tissues and shell are neither homogeneous, nor isotropic. Inclusion of this in the model would be very difficult and would also require significantly better knowledge of crab physiology than is currently available. For now, calculations can be carried out, as in this study, treating organs individually.

- We have, so far, always used the same values for the physical properties of the shell and tissues. However, there is significant seasonal variation in the characteristics of the crab cuticle. Our current Maple sheet is well suited to investigating what effects this variation might have. However, it will require considerable computing time and considerable work tabulating the results for this to be investigated. This is an area that should be investigated since, during seasons when the crab cuticle is "soft" the transmission of waves from the cuticle into the tissues may be significantly enhanced.

- While there are serious concerns about the usefulness of experimentally measuring physical properties of crab tissues, there are so many unknown areas that at least some measurements are worthwhile. The values used for elastic constants of a membrane surrounding a crab ovary should be seen as little better than plausible guesses, given that no data is available for this. The values used for the ovary tissue itself were simply arrived at by assessing what "typical" values are for organ tissues. This can certainly be improved upon. Similarly, the available information on elastic constants of crab cuticle, particularly seasonal variation of this, is extremely limited.

The above items point to future developments of the theory. However, they also point to the need for experiment. These experiments could guide the development of the theory, both by validating the model and by determining which of the above issues have important effects. Approximations which do not have important effects should be kept as the difficulty of removing the approximation is not justified. Thus, experiments which expose targets to incident sound and examine the sound intensity inside fluid and solid filled target spheres would be very helpful in validating the calculations so far carried out. Experiments which

place these spheres in the viscinity of the bottom, or which are carried out on spheroidal targets would help to determine whether it worthwhile to carry out calculations which include the bottom and/or which treat the crab as a spheroid. Finally, experiments using real crabs would help to determine to what extent the results of the simple models resemble the behaviour of a real crab and would help to identify what other considerations must be included in the model. The experiments just described would involve significant technical resources and will require the development of expertise and new experimental techniques. Additionally, experiments to determine the forces necessary to separate the outer membrane from crab ovaries are needed. These latter experiments should be relatively simple.

Finally, the development of the Maple sheets used in these calculations is proceeding well. At the beginning of the project the Maple sheet was only able to solve the differential equations for a fluid-filled model, one frequency at a time, and plot the magnitude of the displacement inside the shell. The Maple sheet now exists in several versions. These are able to:

1. Solve the differential equations for both the fluid filled and the solid filled models.

2. Solve the differential equations at one frequency so that plots or animations of displacements can be generated at that frequency. They can plot animations of total displacement amplitude, longitudinal wave displacement amplitude, shear wave displacement amplitude, radial parts of displacements only, or angular parts of displacements only. These displacements can be plotted in all regions (outside the shell, in the shell material and inside the shell).

3. Solve the differential equations at many frequencies and tabulate parameters of the solutions so that these can be used to generate resonance spectra of various types.

4. Solve at many frequencies and tabulate backscattering amplitudes.

5. Plot energy flux across the shell as a function of time.

6. Solve the differential equations for a model with a solid exterior "tissue".

7. Use a standing wave rather than a traveling wave as the incident wave. It is still restricted to the incident wave being longitudinal.

8. Plot stresses around the surface.

Thus, the Maple sheets are now capable of generating a variety of outputs which can give information on resonances and scattering and which can be used to help a researcher visualize the vibrations of the object in a variety of ways. The multiple versions of the Maple sheet could all be incorporated into a single version where the user chooses what mode it is to run in. This would require some programming of logic and a user interface. This is a straightforward

programming task which could be done by a programmer with no expertise in physics. To be useful to a broad audience the Maple sheets probably need more capability (such as being able to handle cylindrical targets, and display stresses and strains as well as displacements). So it is probably not yet worth it to hire a programmer to build the Maple sheets into a single application. However, building this into an application which is useful to people with no expertise in Maple is a realistic goal in the near future.

References

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Annexes

Solution to the Boundary Value Problem

This section presents, in more detail than was done in [21, Lee-Dadswell, 2008], the calculations which are carried out to determine the sound intensity inside the crab. The presentation here is quite technical and mathematical. Readers wanting a more general overview should refer to [21, Lee-Dadswell, 2008].

We consider an infinite, monochromatic plane wave propagating in the negative z-direction in an infinite, isotropic, homogeneous nonviscous fluid. The wave is incident upon a spherical elastic shell of radius R and thickness d, which contains a third material (Fig. 8). The material inside the shell is modeled as isotropic and homogeneous and is either a nonviscous fluid or an elastic solid. The material of particular interest in this research is the internal tissues of a crab. The outer fluid has bulk modulus K_I, the shell material has bulk and shear moduli K_{II} and μ_{II} respectively and the inner material has bulk and shear moduli K_{III} and μ_{III}.

As usual the displacement in terms of displacement potentials is

$$\mathbf{u}_\gamma = -\nabla\psi_\gamma + \nabla \times \mathbf{\Upsilon}_\gamma, \tag{1}$$

where the subscript $\gamma = \{I, II, III\}$ indicates the region of applicability (see Fig. 8), ψ is the scalar displacement potential and $\mathbf{\Upsilon}$ is the vector displacement potential and

$$\nabla \cdot \mathbf{\Upsilon}_\gamma = 0. \tag{2}$$

The potentials must satisfy the Helmholtz equations

$$\nabla^2\psi_\gamma + k_{\gamma,L}^2\psi_\gamma = 0, \tag{3a}$$
$$\nabla^2\mathbf{\Upsilon}_\gamma + k_{\gamma,T}^2\mathbf{\Upsilon}_\gamma = 0, \tag{3b}$$

where the wave number k_γ is found by $k_\gamma{}^2 = \omega^2\rho_\gamma/(K_\gamma + \frac{4}{3}\mu_\gamma)$, ω is the angular frequency of the incoming waves, ρ_γ is the density of material in region γ, and K_γ and μ_γ are the bulk and shear moduli in region γ. Specifically

$$k_{I,L} \equiv \omega\sqrt{\frac{\rho_I}{K_I}} \tag{4a}$$

$$k_{II,L} \equiv \omega\sqrt{\frac{\rho_{II}}{K_{II} + \frac{4}{3}\mu_{II}}} \tag{4b}$$

$$k_{II,T} \equiv \omega\sqrt{\frac{\rho_{II}}{\mu_{II}}} \tag{4c}$$

$$k_{III,L} \equiv \omega\sqrt{\frac{\rho_{III}}{K_{III} + \frac{4}{3}\mu_{III}}} \tag{4d}$$

$$k_{III,T} \equiv \omega\sqrt{\frac{\rho_{III}}{\mu_{III}}} \tag{4e}$$

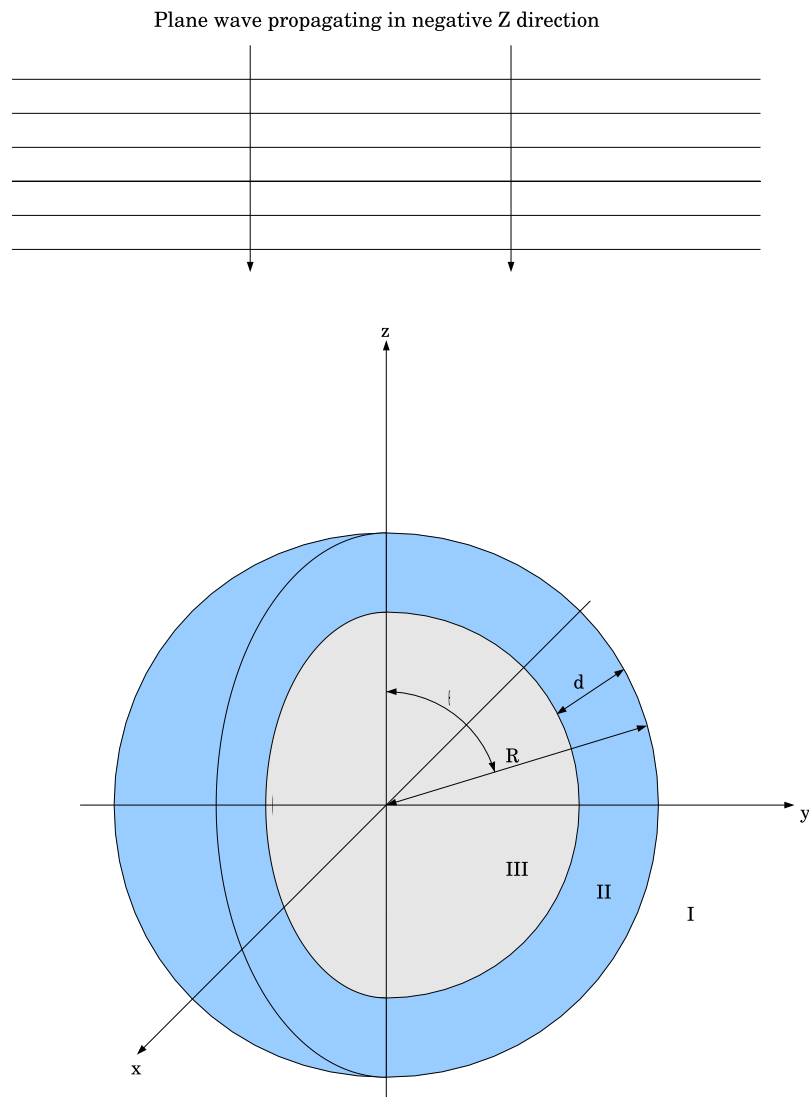Plane wave propagating in negative Z direction

Figure 8: Wave propagating in negative z direction

The incident plane wave, which we take without loss of generality to be of unit magnitude is

$$\psi_i = \exp\left[-i(kz + \omega t)\right] = e^{-i\omega t} \sum_{\ell=0}^{\infty} (2\ell + 1)(-i)^\ell j_\ell(k_{I,L}r) P_\ell(\cos\theta). \quad (5)$$

The scattered wave outside the shell will consist of outgoing spherical waves which can be described by

$$\psi_I = e^{-i\omega t} \sum_{\ell=0}^{\infty} (2\ell + 1)(-i)^\ell A_\ell h_\ell(k_{I,L}r) P_\ell(\cos\theta). \quad (6)$$

The longitudinal part of the wave transmitted into the shell is

$$\psi_{II} = e^{-i\omega t} \sum_{\ell=0}^{\infty} (2\ell + 1)(-i)^\ell [B_\ell j_\ell(k_{II,L}r) + C_\ell n_\ell(k_{II,L}r)] P_\ell(\cos\theta). \quad (7)$$

The longitudinal part of the wave transmitted into the material inside the shell is

$$\psi_{III} = e^{-i\omega t} \sum_{\ell=0}^{\infty} (2\ell + 1)(-i)^\ell F_\ell j_\ell(k_{III,L}r) P_\ell(\cos\theta). \quad (8)$$

Note that the vector displacement potential, $\boldsymbol{\Upsilon}$, only exists in solids and thus only appears in the shell and tissue. By symmetry Υ_r and Υ_θ are zero, therefore the only $\boldsymbol{\Upsilon}$ term needed is Υ_ϕ. The Υ_ϕ in the shell is

$$\Upsilon_{II} = e^{-i\omega t} \sum_{\ell=0}^{\infty} (2\ell + 1)(-i)^\ell [D_\ell j_\ell(k_{II,T}r) + E_\ell n_\ell(k_{II,T}r)] \frac{dP_\ell(\cos\theta)}{d\theta}. \quad (9)$$

The Υ_ϕ in the tissue is

$$\Upsilon_{III} = e^{-i\omega t} \sum_{\ell=0}^{\infty} (2\ell + 1)(-i)^\ell G_\ell j_\ell(k_{III,T}r) \frac{dP_\ell(\cos\theta)}{d\theta}. \quad (10)$$

The above potentials contain the undetermined constants A_ℓ, B_ℓ, C_ℓ, D_ℓ, E_ℓ, F_ℓ and G_ℓ. Thus we need seven equations from the boundary conditions in order to solve for the potentials. To express the boundary conditions we must first know the displacements, stresses, and strains of the system. The radial displacement is

$$u_{\gamma,r} = -\frac{d}{dr}(\psi_\gamma) + \frac{1}{r\sin\theta}\frac{d}{d\theta}(\sin\theta A_\gamma). \quad (11)$$

The tangential displacements are

$$u_{\gamma,\theta} = -\frac{1}{r}\frac{d}{d\theta}(\psi_\gamma) + \frac{1}{r}\frac{d}{dr}(rA_\gamma), \qquad (12a)$$

$$u_{\gamma,\phi} = -\frac{1}{r\sin\theta}\frac{d}{d\phi}(\psi_\gamma). \qquad (12b)$$

The components of the strain tensors are

$$u_{\gamma,rr} = \frac{d}{dr}u_r^{(\gamma)}, \qquad (13a)$$

$$u_{\gamma,\theta\theta} = \frac{1}{r}\frac{d}{d\theta}u_\theta^{(\gamma)} + \frac{u_r^{(\gamma)}}{r}, \qquad (13b)$$

$$u_{\gamma,\phi\phi} = \frac{1}{r\sin\theta}\frac{d}{d\phi}u_\phi^{(\gamma)} + \frac{u_\theta^{(\gamma)}}{r}\cot\theta + \frac{u_r^{(\gamma)}}{r}, \qquad (13c)$$

$$u_{\gamma,r\theta} = \frac{1}{2}\left(\frac{d}{dr}u_\theta^{(\gamma)} - \frac{u_\theta^{(\gamma)}}{r} + \frac{1}{r}\frac{d}{d\theta}u_r^{(\gamma)}\right). \qquad (13d)$$

$$\qquad (13e)$$

The radial stress, given by the generalized Hooke's Law, is

$$\sigma_{rr}^{(\gamma)} = \left(K_\gamma + \frac{4}{3}\mu_\gamma\right)u_{rr}^{(\gamma)} + \left(K_\gamma - \frac{2}{3}\mu_\gamma\right)(u_{\theta\theta}^{(\gamma)} + u_{\phi\phi}^{(\gamma)}). \qquad (14)$$

The deviation in pressure is given in terms of displacement potentials by

$$\begin{aligned}
p' &= \rho\frac{d^2}{dt^2}\psi \\
&= \rho c_\ell^2 \nabla^2 \psi \\
&= -\rho c_\ell^2 k_\ell^2 \psi \\
&= -Kk^2\psi \qquad .
\end{aligned} \qquad (15a)$$

This only applies in regions of fluid. So in region I this gives us

$$p' = -K_I k_I{}^2(\psi_i + \psi_I). \qquad (16)$$

The conditions that must be met at the boundaries between one medium and another are

1. The normal components of the displacements are continuous across the boundaries.

2. The deviation of fluid pressure at a fluid-solid boundary is equal to the normal component of the stress in the solid. Across a solid-solid boundary the normal components of the stress are continuous.

3. Since the fluid is nonviscous the tangential components of shearing stress must vanish at a fluid-solid boundary. At a solid-solid boundary the tangential components of the stress must be continuous across the boundary.

4. The tangential displacement at a solid-solid boundary must be continuous.

For clarity we express the above in terms of the displacement strains and stresses (eqns.11-16)

$$u_{I,r}|_{r=R} = u_{II,r}|_{r=R} \tag{17a}$$

$$u_{II,r}|_{r=R-d} = u_{III,r}|_{r=R-d} \tag{17b}$$

$$p'|_{r=R} = -\sigma_{II,rr}|_{r=R} \tag{17c}$$

$$\sigma_{II,rr}|_{r=R-d} = \sigma_{III,rr}|_{r=R-d} \tag{17d}$$

$$2\mu_2 u_{II,r\theta}|_{r=R} = 0 \tag{17e}$$

$$2\mu_2 u_{II,r\theta}|_{r=R-d} = -2\mu_3 u_{III,r\theta}|_{r=R-d} \tag{17f}$$

$$u_{II,\theta}|_{r=R-d} = u_{III,\theta}|_{r=R-d} \tag{17g}$$

Conditions 1-3 yield 2 equations each because there are two sets of boundaries while condition 4 yields one equation.

These seven equations are now a system of linear equations that can be solved for the constants $\{A_\ell, B_\ell, C_\ell, D_\ell, E_\ell, F_\ell, G_\ell\}$ using standard methods. We have chosen to simply use the algebraic manipulation software Maple, which uses Cramers rule, to solve for the constants. At each frequency examined we carry out the solutions for all values of ℓ from 0 to some ℓ_{max} which we allow to vary with frequency.

Examination of Solutions

We have carried our several tests to verify the correctness of our solutions. One test is the conservation of energy. The energy flux across any closed boundary is

$$\jmath = \oint p'\mathbf{v} \cdot d\mathbf{n} \tag{18}$$

where p' is the deviation from equilibrium pressure, while $\mathbf{v}$ is the local fluid velocity, and $d\mathbf{n}$ refers to an infinitesimal outward directed area element. The symetry of the system makes this simple since the dot product simply picks out the radial component of $\mathbf{v}$

$$v_r = \frac{d}{dt}\frac{d}{dr}\psi \tag{19}$$

and so the energy flux can be represented as

$$\jmath = -Kk^2 \int_0^{2\pi} \int_0^{\pi} d\theta d\phi r^2 \sin(\theta)\psi \left[\frac{d}{dt}\frac{d}{dr}\psi\right] \tag{20}$$

30

The total, time averaged, energy flux across a closed boundary must be zero since there is no damping in the system. We have verified that this is the case for our solutions.

Several comparisons between our results and those in the literature were carried out. We compared our solutions for a fluid interior with the solutions of the equations reported in [26, Matchlup, 1952] and found them to be identical. Taking the inner and outer fluid to be the same, our equations expressing the boundary conditions can be shown to be identical to the system of equations given in [8, Goodman, 1962].

Backscattering cross sections are commonly reported in the literature. Expressing the outgoing (scattered) wave as

$$\psi_I = \frac{e^{ikr}}{r} f(\Omega). \tag{21}$$

where $f(\Omega)$ is the scattering amplitude.

For backscattering ($\theta = 0$ since the wave is moving in the negative z-direction) the Legendre polynomials are simply $P_m(0) = 1$ and so, using (6) and (21), we obtain the backscattering amplitude as

$$f(\Omega_{bs}) = \frac{-i}{k} \sum_{\ell=0}^{\infty} (2\ell + 1) A_\ell (-1)^\ell \quad . \tag{22}$$

Backscattering amplitude for an air-filled steel sphere, as calculated using our Maple sheet, is shown in Fig. 1. It is in good agreement with Figure 8 (a) in [34, Stanton, 1990] for $ka < 10$.

Results

We now examine the solutions to find how the maximum amplitude of motion inside the shell varies with frequency. We expect to see resonant frequencies at which the amplitude as a function of frequency has peaks. These peaks will be the main target of our attention. We use the model described above with the physical constants given in Table 1. The estimation of these values is described in detail in [21, Lee-Dadswell, 2008]

At each frequency we solve for the constants $\{A_\ell, B_\ell, C_\ell, D_\ell, E_\ell, F_\ell, G_\ell\}$ for ℓ from 0 to 8, 12 or 17 depending on the frequency (lower maximum values of ℓ can be used at lower frequencies). We then examine the amplitude of oscillation at points on a grid inside the crab and, for each frequency, select and record the maximum amplitude. These maximum amplitudes vs. frequency are shown for the fluid-filled model in Fig. 2 and for the solid-filled model in Fig. 3.

The amplitudes of the longitudinal waves inside the shell result from the constants F_ℓ. Each peak in Fig. 3 corresponds to a peak in one of the F_ℓ. Accordingly, plotting the various F_ℓ vs. frequency and comparing these with Fig. 3 allows us to determine which modes are resonant at each peak. An example of several F_ℓ vs frequency is shown in Fig. 9 for a restricted range of ka.

31

Physical quantity	Symbol	Value
Bulk modulus of water	K_I	2.05×10^9 N/m^2
Bulk modulus of shell	K_{II}	5×10^9 N/m^2
Shear modulus of shell	μ_{II}	2.05×10^9 N/m^2
Bulk modulus of tissue	K_{III}	3×10^9 N/m^2
Shear modulus of tissue	μ_{III}	1×10^5 N/m^2
Density of water	ρ_I	1000 kg/m^3
Density of shell	ρ_{II}	1000 kg/m^3
Density of tissue	ρ_{III}	1000 kg/m^3
Radius of crab	R	5 cm
Thickness of shell	d	1 mm

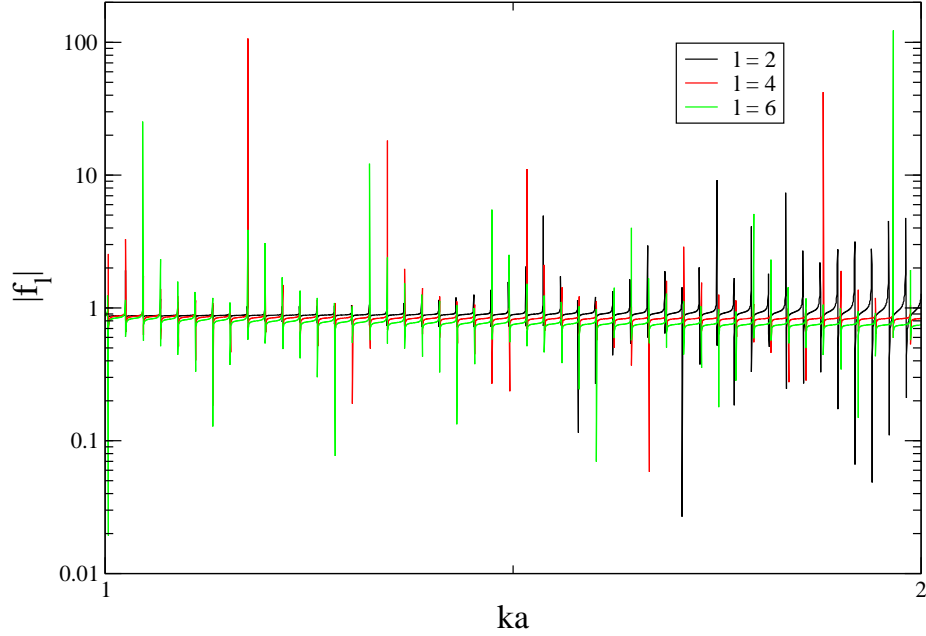Table 1: Values of physical constants used in calculations of sound intensities inside the model crab.

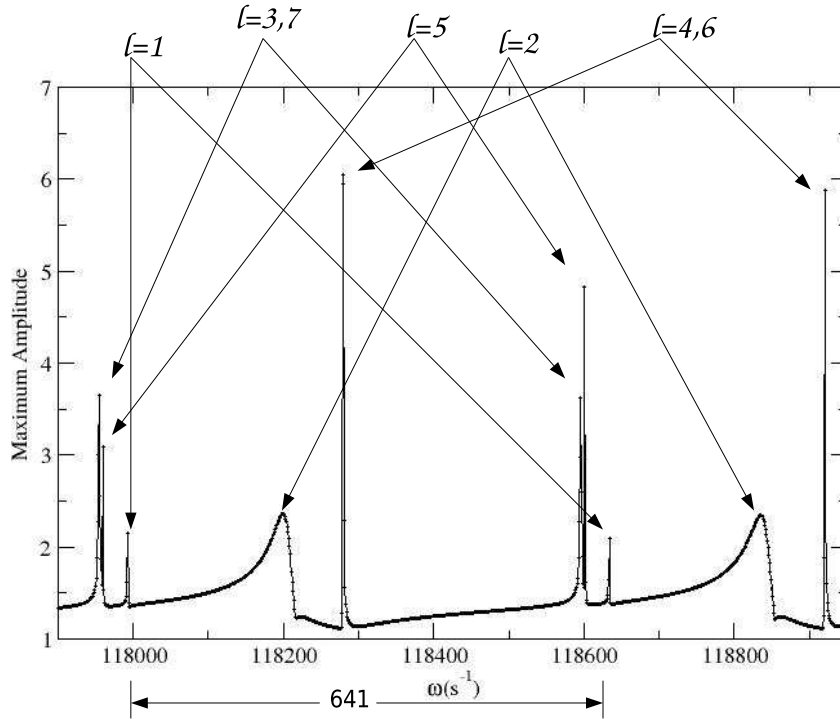Figure 9: $|F_\ell|$ for a sphere with solid interior for $\ell = 4$, 6, 8.

Figure 10: Maximum Amplitude vs ω

The peaks of Fig. 3 can now be identified. Fig. 10 is a repetition of Fig. 4, but with the peaks labeled according to which mode (value of ℓ) is resonant at that frequency. As indicated, the difference in ω between subsequent peaks which result from resonance of the same ℓ is 641 s^{-1}. This gives us an additional check of our results. The frequencies of free vibrations of an elastic sphere can be found via expressions given in [19, Lapwood, 1981]. Using these expressions with our elastic constants we find that the spacing between frequencies of free vibration for constant ℓ of our crab model is 641 s^{-1}, in agreement with the observed spacing of resonances.